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Consensus Tracking of Second Order Multi-agent Systems with Disturbances under Heterogenous Position and Velocity Topologies

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Abstract: In this paper, the consensus tracking problem of second order multi-agent systems with disturbance is studied under heterogenous position and velocity topologies. The cases that the disturbances are generated from linear exosystems and nonlinear exosystems are considered, respectively. For achieving consensus, linear disturbance observer and dynamic-gain-based nonlinear disturbance observer using only the velocity information of the agents are proposed, and then control protocols and sufficient conditions for solving the consensus problem are given. Finally, simulation examples are provided to demonstrate the effectiveness of the proposed disturbance observers and control protocols.

Keywords: Disturbance observer, dynamic gain, heterogenous topologies, multi-agent systems.

1. INTRODUCTION

During the past decades, the cooperative control of multi-agent systems has attracted great attention from various fields such as biology, physics, computer science, applied mathematics and control engineering, due to its wide practical applications in many areas, such as formation control of mobile robots, satellite formation flying, cooperative control of unmanned air vehicles, flocking of social insects, distributed control of communication networks, and rendezvous [1–6], etc. Meanwhile, the consensus of multi-agent systems is an essential cooperation behavior, whose main objective is to design consensus protocols for the agents using only local relative information between neighboring agents to drive the states of all agents reach some common features [7].

The consensus problem for multi-agent systems has been widely studied from various perspectives, and many types of consensus protocols have been proposed in the literature [8–11]. The consensus problem for first-order multi-agent systems has been firstly investigated [12–15]. Specifically, it was shown in [12] that the network connectivity is an influential aspect for reaching consensus. In [13], the authors demonstrated that one of the sufficient and necessary conditions for reaching consensus is that the communication topology graph contains a directed spanning tree. Due to the fact that, in real applications, the dynamics of agents are usually modeled by second-order integrator, especially in some mechanical systems. Thus, the research of consensus problem of second-order multi-agent systems is of great importance [16–20]. For second order multi-agent systems, it was verified in [16] that the existence of a directed spanning tree is only a necessary condition to achieve consensus. In [17], the consensus problem was considered for multi-agent systems with discrete second-order dynamics. In [18], the second order consensus has been investigated for multi-agent systems with sampled data. In [20], some sufficient conditions have been proposed for achieving consensus of second-order multi-agent systems with nonlinear dynamics. Furthermore, the consensus problem has also been studied from various aspects. For example, consensus problem of linear multi-agent systems were studied in [21–23], consensus of agents with nonlinear dynamics were investigated in [24, 25], consensus of multi-agent systems with either time-varying delays or switching topologies were explored in [26, 27], and consensus problem for Markovian jump multi-agent systems have been discussed in [28, 29].

In practical implementation, the dynamics of the agents in a network are always severely affected by various external disturbances due to the complex environment [30–35]. For example, in [31], a disturbance observer-based control approach was proposed for the consensus problem of second-order multi-agent systems with exogenous disturbances, which were assumed to be generated from some

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linear exosystems, and some sufficient conditions for achieving consensus were given by using linear matrix inequality. In [33], consensus problem of second order multi-agent systems with disturbances was further investigated by applying the input-to-state stability and dynamic gain technique when the external disturbances were generated by linear exosystems and nonlinear exosystems, respectively. In [34], robust synchronization problem of dynamical networks with impulsive disturbances was investigated, and some sufficient conditions have been derived to ensure the robust exponential synchronization by using stability analysis for impulsive delay systems.

Note that, most of the aforementioned works mainly focus on studying the consensus problem of multi-agent systems under the assumption that the position and velocity interactions are described by the identical topologies, which will give rise to homogeneous communication networks [36]. However, in reality, the position and velocity interactions among agents are usually transmitted over different network topologies for various reasons, for example, the position and velocity of agents are often measured by different techniques, different sensors are inclined to equip the agents for measuring the position and velocity, respectively [37], and information loss may also lead to the heterogeneous topologies. Unfortunately, up to now, very little attention has been made on the consensus problem of multi-agent systems subject to disturbances when the position and velocity measurements are communicated over different network topologies.

Motivated by the above discussions, in this paper, under the assumption that the position and velocity interactions are characterized by different network topologies, we aim to investigate the consensus tracking problem of second order multi-agent systems with disturbances generated from some linear exosystems and nonlinear exosystems, respectively. The main contributions of this paper are stated as follows: 1) new disturbance observers, which do not need the position information of agents, are proposed to compensate the disturbances generated from heterogeneous linear exosystems and nonlinear exosystems, respectively; 2) the consensus tracking problem of multi-agent systems is investigated for the case that the position and velocity are communicated over different network topologies, which include the problem studied in [33] as a special case when the position and velocity interactions have the identical topologies.

The rest of the paper is organized as follows: Section 2 gives the preliminaries and problem formulation. In Section 3, the disturbance observers and observer-based protocols are proposed. In Section 4, simulation examples are provided to illustrate the designed scheme. Finally, conclusions are shown in Section 6.

2. PRELIMINARIES AND PROBLEM FORMULATION

2.1. Preliminaries

In this subsection, we introduce some basic definitions and results about algebraic graph theory [38]. Let $\mathcal{G} = (V, E, \mathcal{A})$ be a weighted digraph of order N, with a set of nodes $V = \{v_1, v_2, \cdots, v_N\}$, a set of edges $E \subseteq V \times V$, and a weighted adjacency matrix $\mathcal{A} = (a_{ij}) \in R^{N\times N}$. An edge (v_i, v_j) of $\mathcal{G}$ representing that node v_i and v_j can get information with each other. $a_{ij} > 0$ if and only if $(v_j, v_i) \in E$, and $a_{ij} = 0$, otherwise. The set of neighbors of every node v_i is $\mathcal{N}_{v_i} = \{v_j : (v_j, v_i) \in E\}$. A path of $\mathcal{G}$ is an ordered sequence of distinct nodes in V such that any consecutive nodes in the sequence correspond to an edge of the graph. The graph is called connected if there exists a path from v_i to v_j for any two nodes $v_i, v_j \in V$.

The Laplacian matrix $L = (l_{ij}) \in R^{N\times N}$ of a weighted graph is described by

$$l_{ii} = \sum_{j=1, j\neq i}^{N} a_{ij},\ l_{ij} = -a_{ij}, i \neq j.$$

2.2. Problem formulation

Consider a multi-agent system consisting of N followers labeled from 1 to N, and one leader indexed by 0. The dynamics of agent i is governed by

$$\begin{aligned} \dot{x}_i &= v_i, \\ \dot{v}_i &= u_i + d_i,\ \ i = 1, 2, \cdots, N, \end{aligned} \tag{1}$$

where $x_i \in R^n$, $v_i \in R^n$ and $u_i \in R^n$ are the position, the velocity and the control input of agent i, respectively. d_i is the external disturbance that is generated from some linear exosystems or some nonlinear exosystems, which will be specified later.

The dynamics of the leader is described by

$$\begin{aligned} \dot{x}_0 &= v_0, \\ \dot{v}_0 &= a_0(t), \end{aligned} \tag{2}$$

where $x_0, v_0, a_0(t) \in R^n$ with initial state $(x_0(0), v_0(0))$.

With the preceding preparations, we are now ready to introduce our problem.

The problem of consensus tracking for the multi-agent systems consisting of (1) and (2) is to design control protocols u_i such that the N agents in (1) achieve consensus in the sense of $\lim_{t\to\infty} \|x_i(t) - x_0(t)\| = 0, \lim_{t\to\infty} \|v_i(t) - v_0(t)\| = 0, \forall i = 1, 2, \cdots, N$.

To make the problem precise, let $\mathcal{G}^p = (V, E^p, \mathcal{A}^p)$ with $\mathcal{A}^p = (a_{ij})$ and $\mathcal{G}^v = (V, E^v, \mathcal{A}^v)$ with $\mathcal{A}^v = (b_{ij})$ denote the network topologies of position and velocity among all the N agents, respectively. Moreover, the position neighbor set and velocity neighbor set of agent i in $\mathcal{G}^p$ and $\mathcal{G}^v$ are denoted by $\mathcal{N}_i^p = \{v_j : a_{ij} > 0\}$ and $\mathcal{N}_i^v = \{v_j : b_{ij} > 0\}$, respectively.

3. MAIN RESULTS

3.1. Consensus tracking with disturbances generated from heterogeneous linear exosystems

In this subsection, we explore the consensus tracking problem of multi-agent systems (1) and (2) with disturbances that are generated from some heterogeneous linear exosystems of the following form

$$\begin{aligned} &\dot{\xi}_i = E_i \xi_i, \\ &d_i = F_i \xi_i,\ i = 1,2,\cdots,N, \end{aligned} \tag{3}$$

where $\xi_i \in \mathbb{R}^{q_i}$ is the state of the exosystem (3), and the matrix pair (E_i, F_i) is observable.

Now, in order to compensate the disturbances caused by the linear exosystems (3), a disturbance observer is proposed as follows:

$$\begin{aligned} &\dot{z}_i = (E_i - K_i F_i)(z_i + K_i v_i) - K_i u_i, \\ &\hat{\xi}_i = z_i + K_i v_i, \\ &\hat{d}_i = F_i \hat{\xi}_i, \end{aligned} \tag{4}$$

where $z_i \in R^{q_i}$ is the state of the observer (4), K_i is the observer gain matrix that will be determined later, $\hat{\xi}_i$ and $\hat{d}_i$ denote the estimates of ξ_i and d_i, respectively.

By denoting the error between ξ_i and $\hat{\xi}_i$ as

$$e_i = \xi_i - \hat{\xi}_i, \tag{5}$$

we have from (1), (3) and (4) that

$$\dot{e}_i = (E_i - K_i F_i) e_i. \tag{6}$$

Then, we have the following Lemma, which shows that the disturbance observer (4) can asymptotically track the disturbances generated from (3).

Lemma 1: The error system (6) is globally asymptotically stable, if the observer gain matrix K_i is chosen such that the matrix $E_i - K_i F_i$ is Hurwitz.

Remark 1: Note that, compared with the disturbance observer proposed in [33], the observer (4) in this paper does not require the information of x_i and thus is more simple and suitable for applications.

Based on the disturbance observer (4), we propose a consensus tracking protocol as follows

$$\begin{aligned} u_i = &\sum_{j\in\mathcal{N}_i^p} a_{ij}(x_j - x_i) + a_{i0}(x_0 - x_i) + \sum_{j\in\mathcal{N}_i^v} b_{ij}(v_j - v_i) \\ &+ b_{i0}(v_0 - v_i) + a_0(t) - \hat{d}_i, \end{aligned} \tag{7}$$

where the control gain $a_{i0} > 0$ if the ith agent can access the position of the leader and $a_{i0} = 0$ otherwise; analogously, $b_{i0} > 0$ in case that the agent i can obtain the velocity information of the leader and $b_{i0} = 0$ otherwise.

Denote

$$\bar{x}_i = x_i - x_0,\ \bar{v}_i = v_i - v_0, \tag{8}$$

as the tracking error between the followers and the leader. Then, from (1) and (2), one obtains the following error dynamical system:

$$\begin{aligned} &\dot{\bar{x}}_i = \bar{v}_i, \\ &\dot{\bar{v}}_i = u_i + d_i - a_0(t). \end{aligned} \tag{9}$$

Moreover, using (3), (4), (5) and (7), the closed loop system is given as

$$\begin{aligned} \dot{\bar{x}}_i =&\bar{v}_i, \\ \dot{\bar{v}}_i =& \sum_{j\in\mathcal{N}_i^p} a_{ij}(x_j - x_i) + a_{i0}(x_0 - x_i) + \sum_{j\in\mathcal{N}_i^v} b_{ij}(v_j - v_i) \\ &+ b_{i0}(v_0 - v_i) + F_i e_i. \end{aligned} \tag{10}$$

Let

$$\begin{aligned} \bar{x} &= \begin{bmatrix} \bar{x}_1^T & \bar{x}_2^T & \cdots & \bar{x}_N^T \end{bmatrix}^T, \\ \bar{v} &= \begin{bmatrix} \bar{v}_1^T & \bar{v}_2^T & \cdots & \bar{v}_N^T \end{bmatrix}^T, \\ e &= \begin{bmatrix} e_1^T & e_2^T & \cdots & e_N^T \end{bmatrix}^T, \end{aligned}$$

we obtain the global tracking error dynamical system as follows:

$$\begin{aligned} &\dot{\bar{x}} = \bar{v}, \\ &\dot{\bar{v}} = -[(L_p + \triangle_p) \otimes I_n]\bar{x} - [(L_v + \triangle_v) \otimes I_n]\bar{v} + Fe, \\ &\dot{e} = (E - KF)e, \end{aligned} \tag{11}$$

where L_p and L_v are the Laplacian matrices of $\mathcal{G}^p$ and $\mathcal{G}^v$, respectively, and

$$\begin{aligned} &\triangle_p = diag(a_{10}, \cdots, a_{N0}), \\ &\triangle_v = diag(b_{10}, \cdots, b_{N0}), \\ &\mathcal{X} = blockdiag(\mathcal{X}_1, \cdots, \mathcal{X}_N), \mathcal{X} = E, F, K. \end{aligned}$$

Before proceeding further, we cite the following Lemma.

Lemma 2 [39]**:** Suppose that the undirected networks $\mathcal{G}^p$ and $\mathcal{G}^v$ are connected, and there exist $a_{i0} > 0$ and $b_{j0} > 0$ for some $i, j \in \{1, \cdots, N\}$. Then, $L_p + \triangle_p$ and $L_v + \triangle_v$ are two positive definite matrices.

Now, we are ready to state the main result of this subsection.

Theorem 1: For the multi-agent systems consisting of (1)-(3), the problem of consensus tracking can be achieved under the disturbance observer (4) and consensus protocol (7), if the gain matrix K_i is chosen such that $E_i - K_i F_i$ is Hurwitz, and the undirected networks $\mathcal{G}^p$ and $\mathcal{G}^v$ are connected, and there exist $a_{i0} > 0$ and $b_{j0} > 0$ for some $i, j \in \{1, \cdots, N\}$.

Proof: To show that the consensus tracking error $\bar{x}$ and $\bar{v}$ asymptotically converge to zero, a Lyapunov function candidate for system (11) is constructed as follows:

$$V_0 = \frac{1}{2}\bar{x}^T[(L_p + \triangle_p) \otimes I_n]\bar{x} + \frac{1}{2}\bar{v}^T(I_N \otimes I_n)\bar{v} + \varepsilon e^T P e, \tag{12}$$

where $P > 0$ is the solution to the following Lyapunov equation

$$P(E-KF)+(E-KF)^TP=-I, \tag{13}$$

and ε is a positive real number satisfying $\varepsilon \geq \frac{\|(I_N\otimes I_m)F\|^2}{\lambda_{min}(L_v+\triangle_v)}$ with $\lambda_{min}(L_v+\triangle_v)$ is the smallest eigenvalue of $L_v+\triangle_v$. From Lemma 2 and eq. (12), one knows that V_0 is positive definite and radically unbounded.

Then, the derivative of V_0 along system (11) is

$$\begin{aligned}\dot{V}_0(t) =& -\bar{v}^T[(L_v+\triangle_v)\otimes I_n]\bar{v}+\bar{v}^T(I_N\otimes I_n)Fe-\varepsilon e^Te\\ \leq& -\lambda_{min}(L_v+\triangle_v)\|\bar{v}\|^2+\frac{\lambda_{min}(L_v+\triangle_v)}{2}\|\bar{v}\|^2\\ &+\frac{\|(I_N\otimes I_n)F\|^2}{2\lambda_{min}(L_v+\triangle_v)}\|e\|^2-\varepsilon\|e\|^2\\ \leq& -\frac{\lambda_{min}(L_v+\triangle_v)}{2}\|\bar{v}\|^2+\frac{\|(I_N\otimes I_n)F\|^2}{2\lambda_{min}(L_v+\triangle_v)}\|e\|^2\\ &-\varepsilon\|e\|^2\\ \leq& -\frac{\lambda_{min}(L_v+\triangle_v)}{2}\|\bar{v}\|^2-\frac{\varepsilon}{2}\|e\|^2. \end{aligned}\tag{14}$$

In view of (14), we have $\dot{V}_0 \leq 0$ and $\dot{V}_0 \equiv 0$ if and only if $\bar{v}\equiv 0$ and $e\equiv 0$. Thus, from (11), one also has $\bar{x}\equiv 0$. Therefore, it follows from Lasalle's invariance principle that $\lim_{t\to\infty}\|x_i(t)-x_0(t)\|=0, \lim_{t\to\infty}\|v_i(t)-v_0(t)\|=0, \forall i=1,2,\cdots,N$. This completes the proof.

3.2. Consensus tracking with disturbances generated from heterogeneous nonlinear exosystems

In this section, we assume that the disturbance d_i, $i\in\mathscr{F}$, is generated by the following heterogeneous nonlinear exosystem

$$\begin{aligned}&\dot{\xi}_i=E_i\xi_i+\phi_i(\xi_i),\\ &d_i=F_i\xi_i,\ i\in\mathscr{F},\end{aligned}\tag{15}$$

where $\xi_i\in\mathbb{R}^{q_i}$ is the state of the nonlinear exosystem (15), the matrix pair (E_i,F_i) is observable, and $\phi_i(\xi_i)$ is a globally Lipschitz function, i.e., there exists a positive constant $c_{\phi_i}>0$, such that

$$\|\phi_i(\xi_{i1})-\phi_i(\xi_{i2})\|\leq c_{\phi_i}\|\xi_{i1}-\xi_{i2}\|, \tag{16}$$

for any $\xi_{i1},\xi_{i2}\in\mathbb{R}^{q_i}$.

Remark 2: Note that, for the nonlinear exosystems (15), it is assumed that the nonlinear term $\phi_i(\cdot)$ satisfies the condition (16). Nevertheless, this assumption is not restrictive as it might appear to be, since many practical systems satisfy such condition and it is a standard assumption in the literature [40, 41].

In this case, a dynamic-gain-based disturbance observer is proposed as

$$\begin{aligned}\dot{z}_i =&(E_i-K_iF_i)(z_i+K_iv_i)-K_iu_i+\phi_i(z_i+K_iv_i)\\ &+Q_i^{-1}\zeta_ie_i,\\ \dot{\zeta}_i =&e_i^Te_i,\\ e_i =&\xi_i-\hat{\xi}_i,\\ \hat{\xi}_i =&z_i+K_iv_i,\\ \hat{d}_i =&F_i\hat{\xi}_i,\end{aligned}\tag{17}$$

where $z_i\in\mathbb{R}^{q_i}$ is the state of the disturbance observer (17), K_i is the observer gain matrix that will be specified later, ζ_i is the so-called dynamic gain [42], $\hat{\xi}_i$ and $\hat{d}_i$ denote the estimates of ξ_i and d_i, respectively.

Then, one gets

$$\dot{e}_i=(E_i-K_iF_i)e_i+\phi_i(\xi_i)-\phi_i(\hat{\xi}_i)-Q_i^{-1}\zeta_ie_i. \tag{18}$$

Remark 3: Just as stated in Remark 1, in contrast to the disturbance observer in [33] which require the knowledge of both x_i and v_i, the disturbance observer (17) depends on only the information of v_i, and thereby is more suitable for applications. Moreover, for the case of nonlinear exosystem, the estimation error system (18) involves some nonlinear terms $\phi_i(\xi_i)-\phi_i(\hat{\xi}_i)$, and it is difficult to handle such nonlinearity by using constant gain. That is why we resort to dynamic gain technique for constructing the disturbance observer.

Remark 4: It should be noted that, for dealing with the nonlinear terms in (15), some investigations have been presented via fuzzy and piecewise-affine approximation methods in [43] and [44], respectively. Therefore, how to combine the disturbance observer design and fuzzy/piecewise-affine approximation methods for solving the cooperative disturbance rejection problem caused by the nonlinear exosystems is an interesting topic and will be our future considerations.

Then, following the similar ideas of that in [33], we have the following Lemma.

Lemma 3: The estimation error system (18) is globally asymptotically stable if the gain matrix K_i is selected such that the matrix $E_i-K_iF_i$ is Hurwitz, and Q_i is the positive definite matrix solution of the following Lyapunov equation:

$$Q_i(E_i-K_iF_i)+(E_i-K_iF_i)^TQ_i=-I. \tag{19}$$

Moreover, there is a Lyapunov function candidate V_i for (18) satisfying

$$\dot{V}_i\leq-\varepsilon\|e_i\|^2, \tag{20}$$

with $\varepsilon>1$.

Now, the consensus tracking protocol for the ith agent is designed as follows:

$$u_i=\sum_{j\in\mathcal{N}_i^p}a_{ij}(x_j-x_i)+a_{i0}(x_0-x_i)+\sum_{j\in\mathcal{N}_i^v}b_{ij}(v_j-v_i)$$

$$+ b_{i0}(v_0 - v_i) + a_0(t) - \hat{d}_i, \quad (21)$$

where the control gain $a_{i0} > 0$ if the ith agent can access the position of the leader and $a_{i0} = 0$ otherwise; analogously, $b_{i0} > 0$ in case that the agent i can obtain the velocity information of the leader and $b_{i0} = 0$ otherwise.

With the consensus tracking protocol (21), the system (1) can be written as

$$\begin{aligned} \dot{x}_i =& v_i, \\ \dot{v}_i =& \sum_{j\in\mathcal{N}_i^p} a_{ij}(x_j - x_i) + a_{i0}(x_0 - x_i) + \sum_{j\in\mathcal{N}_i^v} b_{ij}(v_j - v_i) \\ &+ b_{i0}(v_0 - v_i) + a_0(t) + F_i e_i. \end{aligned} \quad (22)$$

Furthermore, let

$$\begin{aligned} \bar{x}_i &= x_i - x_0, \\ \bar{v}_i &= v_i - v_0, \\ \bar{x} &= \begin{bmatrix} \bar{x}_1^T & \bar{x}_2^T & \cdots & \bar{x}_N^T \end{bmatrix}^T, \\ \bar{v} &= \begin{bmatrix} \bar{v}_1^T & \bar{v}_2^T & \cdots & \bar{v}_N^T \end{bmatrix}^T, \end{aligned} \quad (23)$$

it follows from (2), (22) and (23) that

$$\begin{aligned} \dot{\bar{x}} &= \bar{v}, \\ \dot{\bar{v}} &= -(H_p \otimes I_m)\bar{x} - (H_v \otimes I_m)\bar{v} + Fe, \\ \dot{e}_i &= (E_i - K_i F_i)e_i + \phi_i(\xi_i) - \phi_i(\hat{\xi}_i) - Q_i^{-1}\zeta_i e_i, \end{aligned} \quad (24)$$

where

$$\begin{aligned} H_p &= L_p + \Delta_p, \\ H_v &= L_v + \Delta_v, \\ F &= blockdiag(F_1, \cdots, F_N). \end{aligned}$$

With the above analysis, the following result is established.

Theorem 2: The problem of consensus tracking of the multi-agent systems consisting of (1), (2) and (15) is solved by the control law (21) based on the disturbance observer (17), if the observer gain K_i is given such that the matrix $E_i - K_i F_i$ is Hurwitz and the undirected networks $\mathcal{G}^p$ and $\mathcal{G}^v$ are connected, and there exist $a_{i0} > 0$ and $b_{j0} > 0$ for some $i, j \in \{1, \cdots, N\}$.

Proof: For the system (24), consider the following Lyapunov function candidate

$$V(t) = \frac{1}{2}\bar{x}^T[(L_p + \Delta_p) \otimes I_m]\bar{x} + \frac{1}{2}\bar{v}^T(I_N \otimes I_m)\bar{v} + \sum_{i=1}^N V_i, \quad (25)$$

where V_i is the Lyapunov function candidate satisfying the inequality (20). According to Lemma 2 and eq. (25), it follows that V is positive definite and radically unbounded.

Then, the derivative of V along the system (24) is given as

$$\begin{aligned} \dot{V}(t) =& -\bar{v}^T[(L_v + \Delta_v) \otimes I_m]\bar{v} + \bar{v}^T(I_N \otimes I_m)Fe + \sum_{i=1}^N \dot{V}_i \\ \le& -\lambda_{min}(L_v + \Delta_v)\|\bar{v}\|^2 + \frac{\lambda_{min}(L_v + \Delta_v)}{2}\|\bar{v}\|^2 \\ &+ \frac{\|(I_N \otimes I_m)F\|^2}{2\lambda_{min}(L_v + \Delta_v)}\|e\|^2 - \varepsilon\sum_{i=1}^N \|e_i\|^2 \\ \le& -\frac{\lambda_{min}(L_v + \Delta_v)}{2}\|\bar{v}\|^2 + \frac{\|(I_N \otimes I_m)F\|^2}{2\lambda_{min}(L_v + \Delta_v)}\|e\|^2 \\ &- \varepsilon\|e\|^2 \\ \le& -\frac{\lambda_{min}(L_v + \Delta_v)}{2}\|\bar{v}\|^2 - \frac{\varepsilon}{2}\|e\|^2, \end{aligned} \quad (26)$$

where ε is selected as $\varepsilon > \max\{1, \frac{\|(I_N \otimes I_m)F\|^2}{\lambda_{min}(L_v + \Delta_v)}\}$.

According to (26), one obtains that $\dot{V} \le 0$ and $\dot{V} \equiv 0$ if and only if $\bar{v} \equiv 0$ and $e \equiv 0$. Therefore, by (24), we also have $\bar{x} \equiv 0$. As a result, it follows from Lasalle's invariance principle that $\lim_{t\to\infty}\|x_i(t) - x_0(t)\| = 0, \lim_{t\to\infty}\|v_i(t) - v_0(t)\| = 0, \forall i = 1, 2, \cdots, N$. This completes the proof.

Remark 5: It is worth noting that, many practical individual systems, especially lots of mechanical systems [45], can be described by second-order dynamics (1), for example, Lagrangian motion dynamics and robotic systems [46], pendulums [47], and harmonic oscillators [48]. On the other hand, the disturbance produced by the linear exosystem (3) is actually harmonic signal with known frequency but unknown amplitude and phase; therefore, the linear exosystem (3) can be employed to depict some kinds of disturbances in engineering applications [49]. Moreover, the nonlinear exosystem (15) is more general, and includes the linear one (3) as a special case; furthermore, it is obviously that one nonlinear exosystem will be capable of creating more exogenous signals than a linear one can [50], which implies that such generalization is of practical signifcance.

Remark 6: In this paper, we extend the results for consensus tracking of second-order multi-agent systems with exogenous disturbance under same position and velocity interaction topologies in [31, 33] to a more general case where the position and velocity interaction among the agents are modeled by different topologies. However, compared with [31, 33], new disturbance observers, which only need the velocity information of the agents, are proposed to compensate the disturbances generated from exosystems. Furthermore, the consensus problem of multi-agent systems with exogenous disturbances under heterogenous position and velocity topologies can be solved by the proposed control protocol, which include the results given in [31, 33] as special cases.

Remark 7: It should be noted that, in this paper, we mainly focus on the consensus tracking problem of second order multi-agent systems under undirected heterogenous position and velocity topologies. In order to show that the proposed design method is also effective when the communication topologies are directed or Markovian switching networks, how to construct some appropriate Lyapunov functions for the cases with directed heterogenous position and velocity topologies or Markovian switching networks are some other interesting topics for future work.

4. NUMERICAL SIMULATIONS

In this section, two simulation examples are presented to illustrate the effectiveness of the proposed disturbance observers and control protocols.

Example 1: Consider the multi-agent systems composed of (1) and (2) with 4 followers and 1 leader. Assume that the disturbances in (1) are generated by the linear exosystems as follows:

$$\begin{aligned} \dot{\xi}_i &= \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix} \xi_i, \\ d_i &= \begin{bmatrix} 1 & 0 \end{bmatrix} \xi_i, \quad i = 1,2,3,4. \end{aligned} \tag{27}$$

and the leader with varying velocity is modeled by

$$\begin{aligned} \dot{x}_0 &= v_0, \\ \dot{v}_0 &= \sin(t). \end{aligned} \tag{28}$$

From (27), one knows that $E_i = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}, F_i = \begin{bmatrix} 1 & 0 \end{bmatrix}$, and (E_i, F_i) is observable. For illustration, assume that the communication topologies of position and velocity among the leader and followers are depicted in Fig. 1. From Fig. 1, one knows that $\mathcal{G}^p$ and $\mathcal{G}^v$ are connected, $a_{10} > 0$ and $b_{20} > 0$. Therefore, by Theorem 1, the proposed disturbance observer (4) and control protocol (7) can solve the consensus tracking problem of the multi-agent systems composed of (1) and (2) with the disturbance generated from linear exosystems (27). In the simulation, set the observer gain as $K_1 = \begin{bmatrix} 1.3522 & 0.4142 \end{bmatrix}^T, K_2 = \begin{bmatrix} 1.3944 & 0.2361 \end{bmatrix}^T, K_3 = \begin{bmatrix} 1.4049 & 0.1623 \end{bmatrix}^T, K_4 = \begin{bmatrix} 1.4088 & 0.1231 \end{bmatrix}^T$. The simulation results are shown in Fig.2 and Fig. 3. The initial states of the linear exosystems, and the initial positions and velocities of agents are chosen randomly from $[-5, 5]$. The initial states of the disturbance observer are chosen as 0. From Fig.2 and Fig. 3, it can be seen that, when the position and velocity measurements are communicated over different network topologies, the proposed disturbance observer and control protocol can make the follower agents' positions and velocities reach consensus with the leader agent's position and velocity, respectively.

Example 2: In this example, we consider the consensus tracking problem of multi-agent systems (1)-(2), and

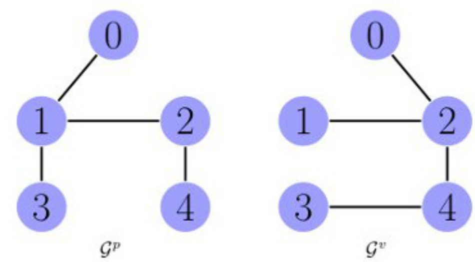

Fig. 1. Communication topologies of position and velocity among the leader and followers.

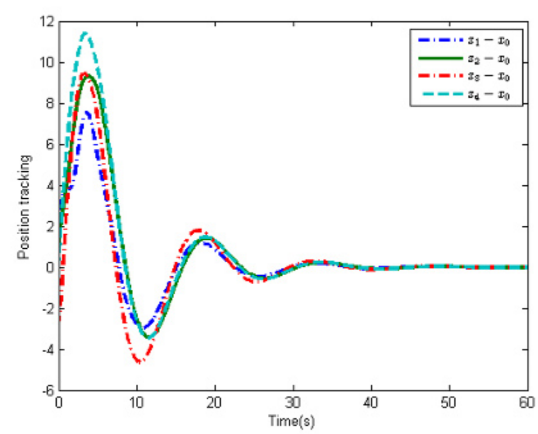

Fig. 2. Positions consensus among the leader and followers under (4) and (7).

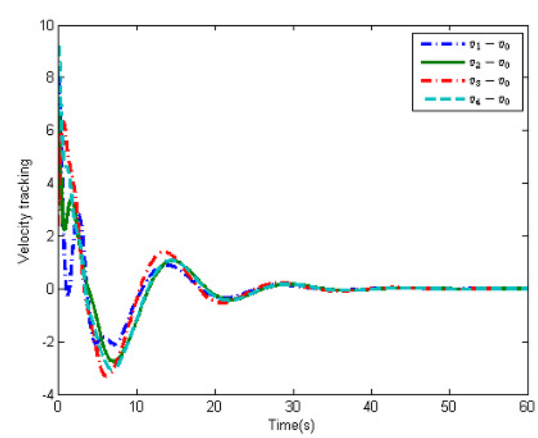

Fig. 3. Velocities consensus among the leader and followers under (4) and (7).

the communication graph of the position and velocity among the leader and followers are also shown in Fig. 1. Here, different from Example 1, the disturbances are produced from the following nonlinear exosystems

$$\begin{bmatrix} \dot{\xi}_{i1} \\ \dot{\xi}_{i2} \end{bmatrix} = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix} \begin{bmatrix} \xi_{i1} \\ \xi_{i2} \end{bmatrix} + \begin{bmatrix} 0 \\ i \sin(\xi_{i1}) \end{bmatrix},$$

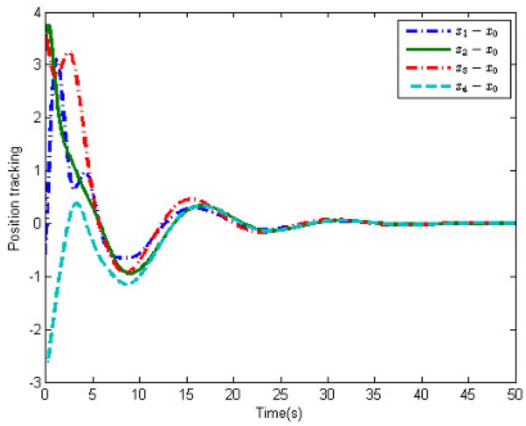

Fig. 4. Position Velocities consensus among the leader and followers under (17) and (21).

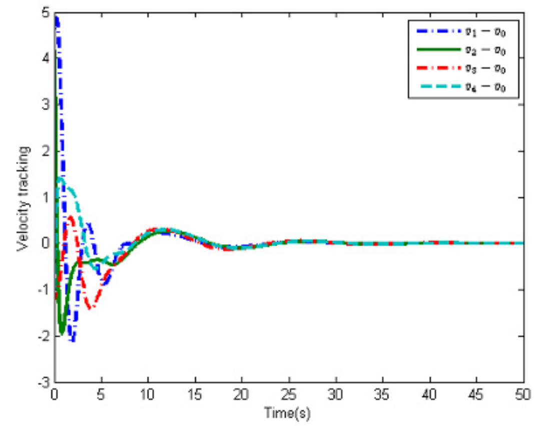

Fig. 5. Velocities consensus among the leader and followers under (17) and (21).

$$d_i = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}\xi_{i1}\\ \xi_{i2}\end{bmatrix}, \quad i = 1,2,3,4. \tag{29}$$

Now, according to Lemma 3 and Theorem 2, under heterogenous position and velocity topologies, the multi-agent systems (1)-(2) with disturbances generated by nonlinear exosystems (29) can achieve consensus. In the simulation, select the same observer gain matrix K_i as in Example 1, and then solve the Lyapunov equation (19), we obtain the matrix gain Q_i as

$$Q_1 = \begin{bmatrix}0.8927 & -0.5\\ -0.5 & 1.1093\end{bmatrix}, Q_2 = \begin{bmatrix}0.7595 & -0.25\\ -0.25 & 0.8352\end{bmatrix},$$
$$Q_3 = \begin{bmatrix}0.7311 & -0.1667\\ -0.1667 & 0.7676\end{bmatrix}, Q_4 = \begin{bmatrix}0.7207 & -0.125\\ -0.125 & 0.7419\end{bmatrix}.$$

The initial states of the nonlinear exosystems and the multi-agent systems are chosen randomly from the interval $[-5,5]$, while the initial states of the disturbance observer (17) are selected as 0. The simulation results are presented in Fig. 4 and Fig. 5, which indicate that the consensus tracking problem of multi-agent systems (1) and (2) in the presense of disturbances generated from nonlinear exosystems (29) is achieved via the proposed disturbance observer (17) and control protocol (21).

5. CONCLUSIONS

Under heterogenous position and velocity topologies, the consensus tracking problem of second order multi-agent systems was considered in the presense of disturbances generated from linear exosystems and nonlinear exosystems, respectively. For each case, disturbance observer was presented, which is independent of the position measurement of the agents. Furthermore, with the aid of disturbance observers, control protocols and sufficient conditions for achieving consensus were proposed. The effectiveness of the proposed results were verified through simulation results.

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